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14. ABSTRACT

This work considers the problem of estimating directions of arrival (DOAs) from sensor-array data when the positions of the sensors are not precisely known. The proposed algorithm is an extension of a previously developed approach based on weighted subspace fitting. The new algorithm shows improved robustness with respect to sensor position perturbations, particularly when estimating the DOAs of weak sources. The robust algorithm treats the sensor position perturbations as unknown deterministic parameters and estimates them jointly with the DOAs. The distances between each sensor in the perturbed array were constrained to equal their nominal values. The algorithm adds one source at a time to the model until the amplitude of an identified source is below a threshold. For each source added to the model, the algorithm performs a one-dimensional grid searches followed by multidimensional optimization over a small region of parameter space. This method is effective for estimating a large number of parameters.

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Robust Direction Finding

Final Report for Grant N00014-04-1-0151

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Abstract

This work considers the problem of estimating directions of arrival (DOAs) from sensor-array data when the positions of the sensors are not precisely known. The proposed algorithm is an extension of a previously developed approach based on weighted subspace fitting. The new algorithm shows improved robustness with respect to sensor position perturbations, particularly when estimating the DOAs of weak sources. The robust algorithm treats the sensor position perturbations as unknown deterministic parameters and estimates them jointly with the DOAs. The distances between each sensor in the perturbed array were constrained to equal their nominal values. The algorithm adds one source at a time to the model until the amplitude of an identified source is below a threshold. For each source added to the model, the algorithm performs a one-dimensional grid searches followed by multidimensional optimization over a small region of parameter space. This method is effective for estimating a large number of parameters.

1 Introduction

The performance of array processing algorithms is degraded when the sensor positions are perturbed from their nominal values. Various methods have been proposed to deal with this problem. Gilbert and Morgan [1] considered the calculation of beamforming weights. They showed that weights required to produce a very narrow main beam gave a severely degraded beampattern when the sensor positions were perturbed. They derived a formula to calculate the narrowest possible main beam subject to a constraint on the degradation caused by sensor position perturbation.

Schultheiss and Ianiello [2] considered the effect of sensor postion perturbations on the problem of estimating the range and bearing of a single source. They showed that, to first order, conventional beamforming, which is optimal when there is no perturbation, remains optimal for small array perturbations. However, this result is valid only for a single source.

In this paper we propose a robust algorithm for estimating the directions of arrival of multiple sources based on weighted subspace fitting. This is an extension of the WSF algorithm presented in [3]. In addition to presenting a cost function, [3] showed how to obtain initial DOA estimates, determine the number of sources in the data, and handle multiple clusters of closely spaced, non-resolvable sources. However, the algorithm in [3] assumed that the sensor positions were precisely known, and its performance is degraded when there is sensor position perturbation. In this paper, we remove to some degree this degradation by jointly estimating the postion perturbations, which are considered to be deterministic unknown parameters, and the DOAs.

There is related work in references [4]-[6]. In [4] a WSF cost function was derived based on the asymptotic statistics of perturbed eigenvectors, whereas the WSF cost function used in this paper was derived from a subspace perturbation expansion [3]. References [5] and [6] use statistical models for array perturbations, including sensor position errors and sensor gain and phase errors. A WSF cost function is derived in which the weights are calculated to minimize the DOA estimation error variance with respect to both the additive noise in the data and the array perturbations.

2 Data Model

The model for the noise-free signal at a single frequency is

$$\mathbf{Y} = \mathbf{A}(\boldsymbol{\theta}_0, \mathbf{x}, \mathbf{y}, \mathbf{z})\mathbf{S} = [\, \mathbf{a}_1 \quad \cdots \quad \mathbf{a}_r \,]\, \mathbf{S} \tag{1}$$

where $\mathbf{Y}$ is $m \times n$, $\mathbf{A}$ is $m \times r$, and $\mathbf{S}$ is $r \times n$. In this application, m is the number of sensors, n is the number of snapshots of array data, and r is the number of narrowband signals impinging on the array. The vector of possible DOAs is $\boldsymbol{\theta}$ and $\boldsymbol{\theta}_0$ denotes the actual DOAs. The vectors $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{z}$ contain the coordinates (positions) of the sensors.

In this paper we consider a nominally uniform linear array of sensors whose coordinates are:

$$\mathbf{x}_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \mathbf{y}_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \mathbf{x}_0 = \begin{bmatrix} 0 \\ d \\ 2d \\ \vdots \\ (m-1)d \end{bmatrix}. \tag{2}$$

Thus, the sensors lie along the z axis with a spacing of $d = 1/2$ between each sensor. The units of d are wavelengths corresponding to an array design frequency f_0.

The columns of $\mathbf{A}$ are the array manifold vectors, and they take the following functional form:

$$\mathbf{a}_i = e^{-j2\pi\alpha(\mathbf{z}\cos\theta_i + \mathbf{x}\sin\theta_i)} \tag{3}$$

where α is the signal frequency expressed as a fraction of f_0, the array design frequency. In other words, the actual signal frequency is αf_0. Note that for simplicity, all of the analysis in this paper takes place in the xz plane (all y components equal zero). The approach can be extended to a full three-dimensional analysis.

We rewrite the array manifold vectors in terms of two vectors

$$\boldsymbol{\delta}\mathbf{z} = \begin{bmatrix} \delta z_1 \\ \vdots \\ \delta z_m \end{bmatrix} \text{ and } \boldsymbol{\delta}\mathbf{x} \begin{bmatrix} \delta x_1 \\ \vdots \\ \delta x_m \end{bmatrix} \tag{4}$$

that represent perturbations in the z and x components of sensor position, respectively. We assume that all sensor positions are referenced to the first sensor (i.e. there is no perturbation associated with the first sensor). Thus, the actual sensor locations are as follows:

$$\mathbf{x}_a = \begin{bmatrix} 0 \\ \delta x_1 \\ \delta x_2 \\ \vdots \\ dx_{m-1} \end{bmatrix}, \mathbf{y}_a = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \mathbf{z}_a = \begin{bmatrix} 0 \\ d + \delta z_1 \\ 2d + \delta z_2 \\ \vdots \\ (m-1)d + \delta z_{m-1} \end{bmatrix}. \tag{5}$$

The nominal and perturbed sensor locations for the first three sensors of a line array are shown in Fig. 1. We treat the position perturbations in the x direction (perpendicular to the line of the array) as free variables. The perturbations in the z direction (along the line of the array) are calculated so as to keep the spacing between the sensors equal to d. This is expressed with the following constraint equations:

$$(\delta x_i - \delta x_{i-1})^2 + (d + \delta z_i - dz_{i-1})^2 = d^2, \ i = 1, \cdots, m-1. \tag{6}$$

These equations can be solved for δz in terms of δx, with $\delta x_0 = \delta z_0 = 0$, as follows

$$\delta z_i = \delta z_{i-1} + \sqrt{d^2 - (\delta x_i - \delta x_{i-1})^2} - d, \ i = 1, \cdots, m-1. \tag{7}$$

The position vectors for the actual sensor locations are

$$\mathbf{x} = \boldsymbol{\delta}\mathbf{x}_a, \ \mathbf{y} = \mathbf{0}, \ \mathbf{z} = \mathbf{z}_0 + \boldsymbol{\delta}\mathbf{z}_a. \tag{8}$$

Thus, the array manifold vectors can be written in terms of the position perturbation vectors as follows:

$$\mathbf{a}_i = e^{-j2\pi\alpha((\mathbf{z}_0 + \boldsymbol{\delta}\mathbf{z})\cos\theta_i + \boldsymbol{\delta}\mathbf{x}\sin\theta_i)} \tag{9}$$

The SVD of $\mathbf{Y}$ is

$$\mathbf{Y} = [\mathbf{U}_1 \quad \mathbf{U}_2] \begin{bmatrix} \boldsymbol{\Sigma}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1^H \\ \mathbf{V}_2^H \end{bmatrix} \tag{10}$$

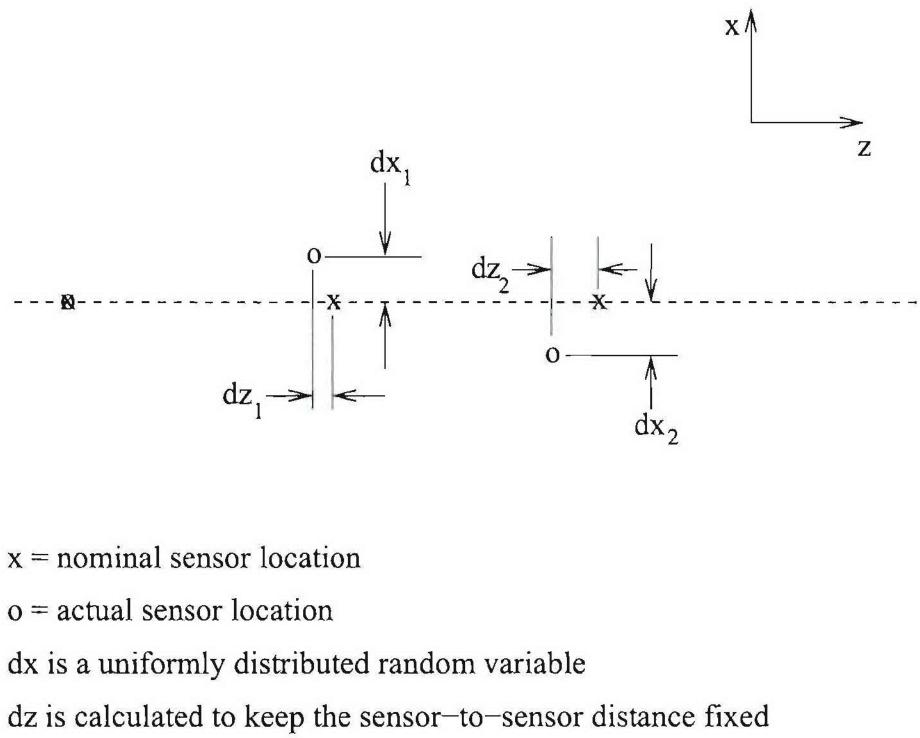

Figure 1: Nominal and perturbed sensor locations for a 3-element array.

where $\mathbf{U}_1$ has r columns. The columns of $\mathbf{U}_1$ and $\mathbf{A}(\boldsymbol{\theta}_0)$ span the same subspace, and therefore columns of $\mathbf{U}_2$ are orthogonal to columns of $\mathbf{A}(\boldsymbol{\theta}_0)$.

The observed (noisy) data is

$$\tilde{\mathbf{Y}} = \mathbf{Y} + \mathbf{N} \tag{11}$$

where the elements of $\mathbf{N}$ are taken to be zero mean i.i.d. complex Gaussian random variables with variance σ^2 (real and imaginary parts are uncorrelated). The SVD of $\tilde{\mathbf{Y}}$ is

$$\tilde{\mathbf{Y}} = [\, \tilde{\mathbf{U}}_1 \quad \tilde{\mathbf{U}}_2 \,] \begin{bmatrix} \tilde{\boldsymbol{\Sigma}}_1 & \mathbf{0} \\ \mathbf{0} & \tilde{\boldsymbol{\Sigma}}_2 \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{V}}_1^H \\ \tilde{\mathbf{V}}_2^H \end{bmatrix}$$

where $\tilde{\mathbf{U}}_1$ has r columns.

The subspace-fitting criterion used here is based on the fact that, in the noise-free case,

$$\mathbf{U}_2\mathbf{U}_2^H\mathbf{A}(\boldsymbol{\theta}_0, \boldsymbol{\delta}\mathbf{x}_a, \mathbf{0}, \mathbf{z}_0 + \boldsymbol{\delta}\mathbf{z}_a) = \mathbf{0}.$$

With noisy data the previous expression will not equal zero and we look for the parameter vector $\hat{\boldsymbol{\theta}}$ that minimizes the equation error:

$$\hat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} \left[\min_{\boldsymbol{\delta}\mathbf{x}} \|\tilde{\mathbf{U}}_2\tilde{\mathbf{U}}_2^H\mathbf{A}(\boldsymbol{\theta}, \boldsymbol{\delta}\mathbf{x}, \mathbf{0}, \mathbf{z}_0 + \boldsymbol{\delta}\mathbf{z})\|_{\mathbf{W}}^2 \right] \tag{12}$$

where (7) is used to calculate $\boldsymbol{\delta}\mathbf{z}$ as a function of $\boldsymbol{\delta}\mathbf{x}$, and the norm is defined as

$$\|\cdot\|_{\mathbf{W}}^2 = \operatorname{vec}(\cdot)^H\mathbf{W}\,\operatorname{vec}(\cdot)$$

and $\mathbf{W}$ is a weight matrix.

The weight matrix $\mathbf{W}$ is calculated from the statistics of the vectorized cost function evaluated at the true parameters [3]. Note that the perturbation of the cost function at the true parameters is due only to the additive noise. Thus, the matrix $\mathbf{W}$ in this paper is the same as that used in [3]. After substituting in the weight matrix, the cost function may be simplified to

$$C(\boldsymbol{\theta}, \boldsymbol{\delta}\mathbf{x}) = \|\tilde{\mathbf{U}}_2^H \mathbf{A}(\boldsymbol{\theta})(\tilde{\mathbf{U}}_1^H \mathbf{A}(\boldsymbol{\theta}))^{-1}\hat{\boldsymbol{\Sigma}}_1\|_F^2, \tag{13}$$

where $\mathbf{A} = \mathbf{A}(\boldsymbol{\theta}, \boldsymbol{\delta}\mathbf{x}, \mathbf{0}, \mathbf{z}_0 + \boldsymbol{\delta}\mathbf{z})$, $\hat{\boldsymbol{\Sigma}}_1 = (\tilde{\boldsymbol{\Sigma}}_1^2 - \hat{\sigma}^2\mathbf{I})^{0.5}$, and $\hat{\sigma}^2$ is the average of the squared singular values in $\tilde{\boldsymbol{\Sigma}}_2$. Finally, the subscript 'F' in (13) denotes the Frobenious matrix norm.

3 Review of Previous Algorithm

The steps of the original, nonrobust algorithm [3] are reviewed in this section. The modifications which are needed to give robustness to sensor position perturbations are given in the following section. In the original algorithm, the cost function is given by (13) with the position perturbation parameters fixed at zero. The algorithm proceeds as follows:

Step 1

Set $r = 1$ (look for one source) and plot the reciprocal of the cost function on a grid of points to find the maximum. The angle θ that maximizes the reciprocal of the cost function is called $\hat{\theta}_1$.

Step 2

To search for the second source, set $r = 2$ and

$$\boldsymbol{\theta} = \begin{bmatrix} \hat{\theta}_1 \\ \theta_2 \end{bmatrix}$$

where $\hat{\theta}_1$ is fixed at its value from the first step. The reciprocal of the cost function is plotted as a function of θ_2. The value of θ_2 that maximizes the reciprocal of the cost function is called $\hat{\theta}_2$.

Step 3

With

$$\boldsymbol{\theta} = \begin{bmatrix} \hat{\theta}_1 \\ \hat{\theta}_2 \end{bmatrix}$$

as an initialization, $C(\boldsymbol{\theta})$ is minimized (e.g. using the Matlab Optimization Toolbox), and the signal powers and noise powers are estimated [3].

Step 4

This process of adding one source at a time is repeated until the SNR estimated for any source is less than a user-specified threshold SNR. The initial value for each new source is obtained using a one-dimensional grid search of the cost function with all previously found DOAs fixed.

After the initial value of the new source is found, the cost function is minimized with respect to all of the DOAs. This minimization is done with tight upper and lower bounds on the DOAs, centered on their current values.

Because of the weighting in the cost function, the sources are found roughly in the order of their powers, from strongest to weakest. This ordering is occasionally altered when there are sources of nearly equal power.

4 Robust Algorithm

Simulation results show that the original algorithm has some robustness to sensor perturbations. The main effects of these perturbations are that the variances of the DOA estimates increase and the estimated SNRs decrease due to the mismatch between the array manifold model used to calculate the SNR and the actual (perturbed) array manifold. The original algorithm gives reasonable estimates for all but the weakest sources. Thus, we can use the original algorithm with a high SNR threshold to initialize the robust algorithm.

For example, in the simulations shown in the next section, the original algorithm uses an SNR threshold of -10 dB when there are no sensor position perturbations. In order to deal with sensor perturbations, the threshold is set to 10 dB, After sources above 10 dB have been estimated, the remaining sources are estimated jointly with the perturbation parameters $\boldsymbol{\delta}\mathbf{x}$. The grid searches for each new DOA are done with the $\boldsymbol{\delta}\mathbf{x}$ parameters fixed at their current values. The minimizations are done with respect to all DOA and perturbation parameters.

The robust algorithm treats the actual sensor position perturbations as unknown deterministic parameters. The initial guesses for the perturbation parameters are all zero, and tight upper and lower bounds (± 0.1) are used for each element of $\boldsymbol{\delta}\mathbf{x}$. The perturbation parameters are expressed as a fraction of the sensor spacing, d. We use the same sequential precedure as before, adding one source at a time.

5 Simulation Example

In this section we consider a challenging simulation example consisting of seven moving sources, some strong and others weak, at normalized frequency 0.4. The array is a 48-element uniform linear array.

Fig. 2 shows a time-bearing plot of the estimated sources obtained with the original algorithm; the data are generated with no array perturbation. The colorbar gives the estimated signal-to-noise ratios of each estimated source. The given data contained 1800 snapshots from the array and these were processed in blocks of 15 snapshots. In this example the number of sources was not assumed to be known in advance. For each matrix processed, sources were estimated one at a time. Each time a new source was estimated, the SNR associated with all of the sources was also estimated. Additional sources were added to the model until the SNR associated with one of the estimated sources was less than -10 dB.

In the remaining examples, the data were generated with some amount of array perturbation. The $\boldsymbol{\delta}\mathbf{x}$ parameters specifying the perturbed array were generated as iid random variables uniformly distributed on the interval $[-\Delta, \Delta]$, where Δ is specified as a fraction of the inter-sensor spacing, d. Fig. 3 shows the result of the original algorithm processing data generated with $\Delta = 0.02$. Notice that the weak sources near 0 dB are not well estimated (compare with Fig. 2).

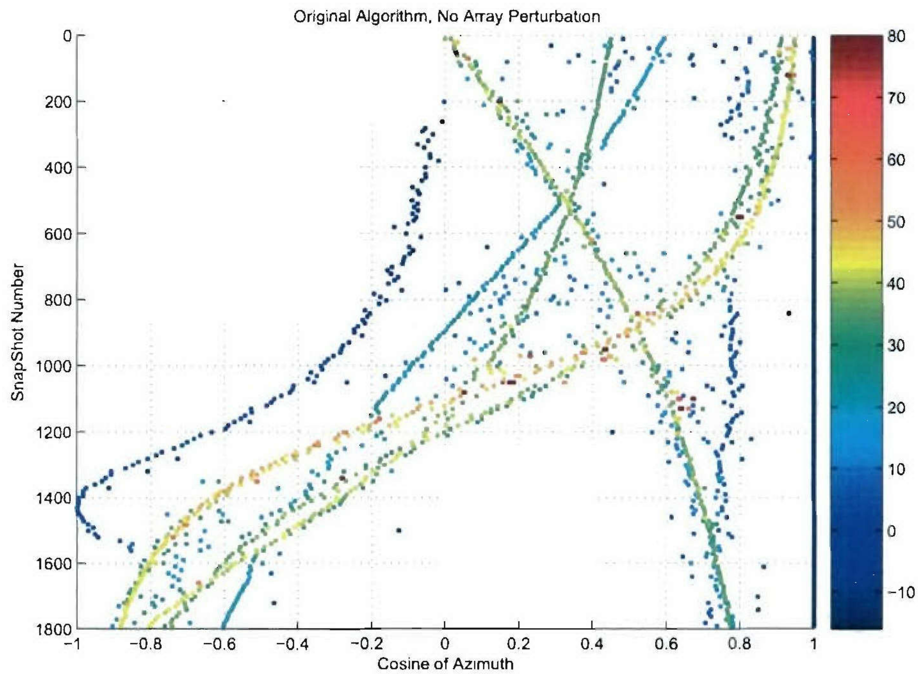

Figure 2: Time-bearing plots for a seven-source simulation. Color bar indicates estimated signal-to-noise ratio in dB. Original algorithm. No array perturbation.

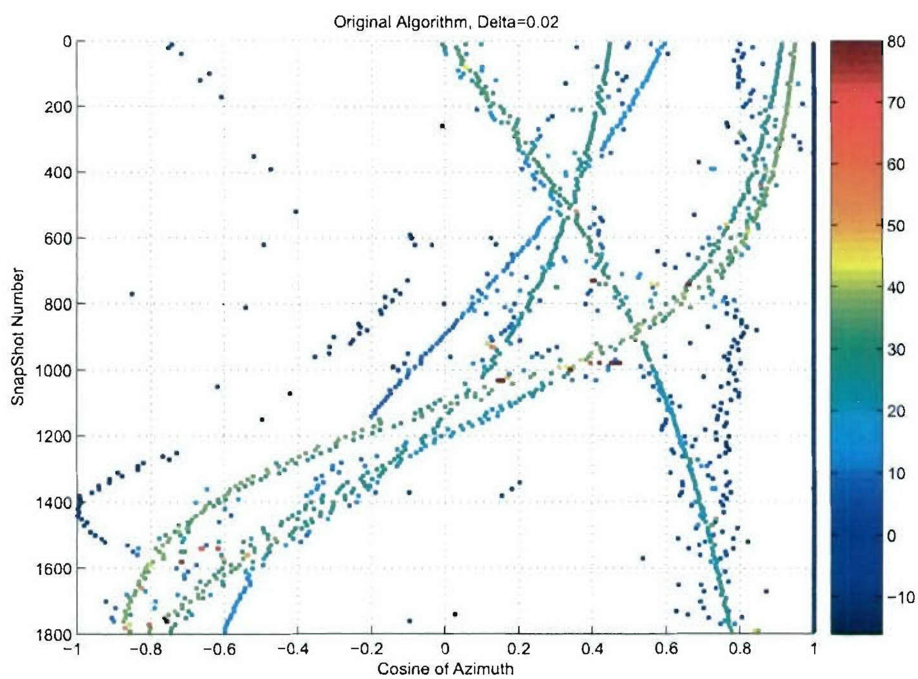

Figure 3: Time-bearing plot for seven-source example. Original algorithm. Data generated with array perturbation, $\Delta = 0.02$.

Fig. 4 shows the result of the original algorithm processing data generated with $\Delta = 0.04$. The weak sources are no longer visible.

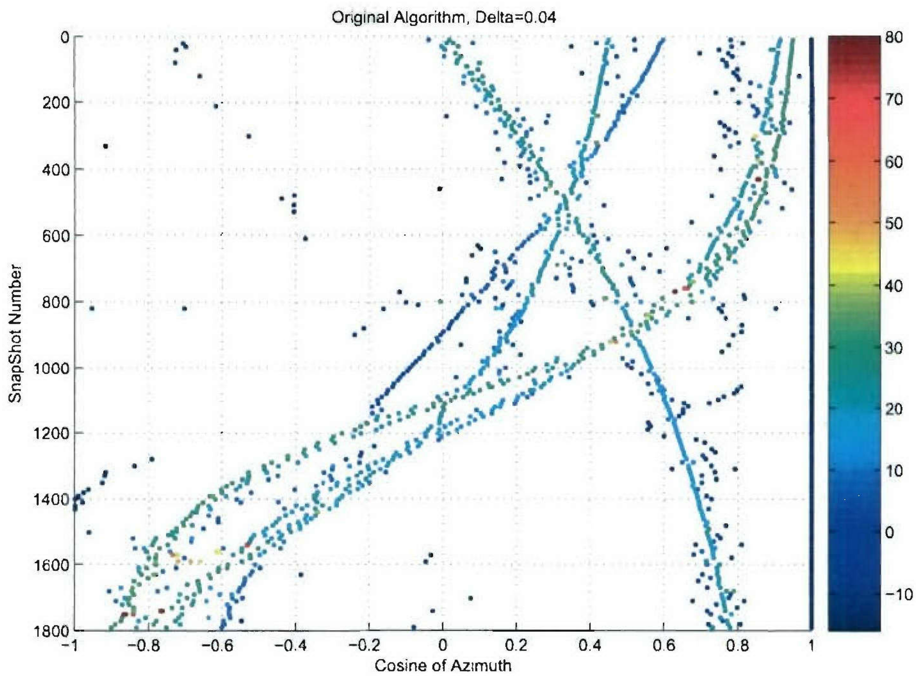

Figure 4: Time-bearing plot for seven-source example. Original algorithm. Data generated with array perturbation, $\Delta = 0.04$.

Figs. 5 and 6 show the results of the robust algorithm processing data generated with $\Delta = 0.02$ and 0.04, respectively. Although there is some degradation in the estimation of the weak sources, there is a noticeable improvement over the original algorithm (Figs. 3 and 4).

6 Summary

The sensitivity of the WSF algorithm described in [3] to sensor position perturbations can be reduced by estimating these perturbations along with the DOAs. Using this approach, one is able to incorporate constraints on the perturbations. In this paper the distances between each sensor in the perturbed array were constrained to equal their nominal values. The sequential, add one source at a time, procedure of one-dimensional grid searches followed by multidimensional optimization over a small region of parameter space is an effective method for estimating a large number of parameters.

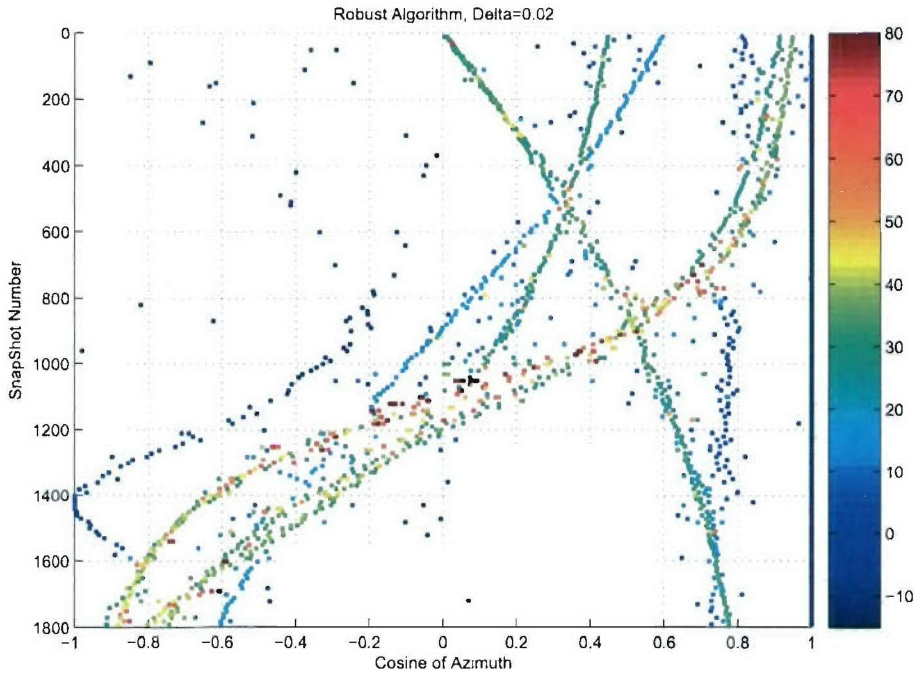

Figure 5: Time-bearing plot for seven-source example. Robust algorithm. Data generated with array perturbation, $\Delta = 0.02$.

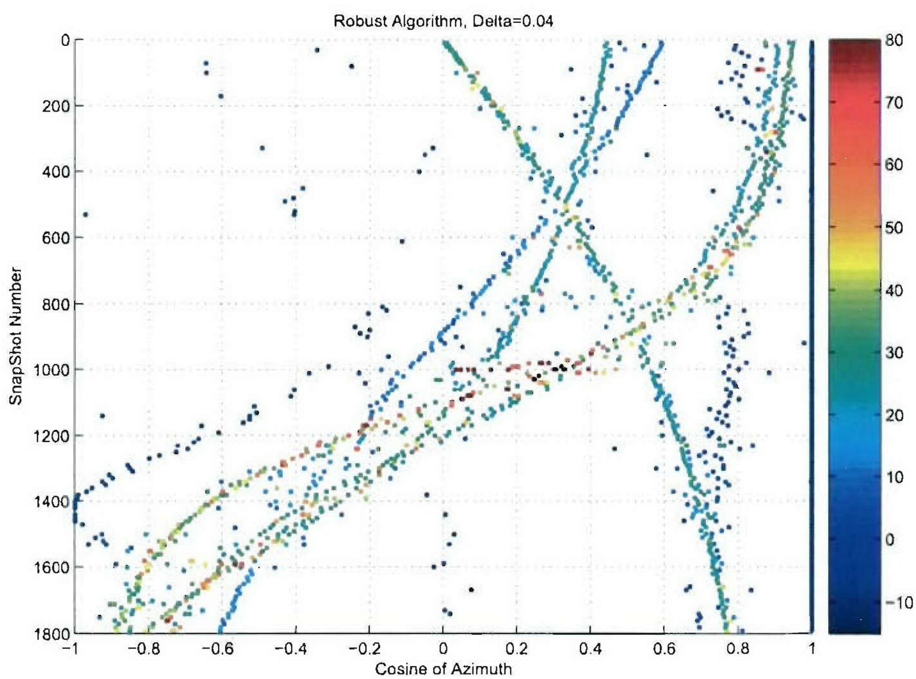

Figure 6: Time-bearing plot for seven-source example. Robust algorithm. Data generated with array perturbation, $\Delta = 0.04$.

References

[1] E. N. Gilbert and S. P. Morgan, "Optimum Design Of Directive Antenna Arrays Subject to Random Variations," The Bell System Technical Journal, pp. 637-663, May 1955.

[2] P. Schultheiss and J. Ianniello, "Optimum Range and Bearing Estimation with Randomly Perturbed Arrays," J. Acoust. Soc. Amer., 68(1): pp.167–173, July, 1980.

[3] R.J. Vaccaro, P. Majumdar, and N. Owsley, "An Approach to Direction Finding Based on a Subspace Perturbation Expansion," in*Proc. 37th Asilomar Conference on Signals, Systems, and Computers*, Pacific Grove, CA, pp. 817-821, November 9-12, 2003.

[4] M. Viberg and B. Ottersten, "Sensor Array Processing Based on Subspace Fitting," *IEEE. Trans. Signal Processing*, pp. 1110–1121, May 1991.

[5] B. Wahlberg, B. Ottersten, and M. Viberg, "Robust Signal Parameter Estimation in the Presence of Array Perturbations," in *Proc. IEEE ICASSP*, Toronto, Ont. Canada, pp. 3277–3280, 1991.

[6] M. Jansson, A.L. Swindlehurst, and B. Ottersten, "Weighted Subspace Fitting for General Array Error Models," *IEEE. Trans. Signal Processing*, pp. 2484--2498, September 1998.